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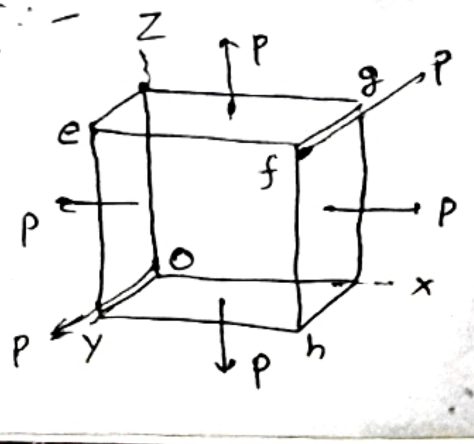
Q What are the elastic constant. Obtain the relation between them. Show that the value of Poisson's ratio lies b/n 0.5 and -1.

Ans These constants are the constants of proportionality in Hook's law and they may be. Young's modulus (Y), bulk modulus (K), Modulus of rigidity (η) & Poisson's ratio (σ).

Upto the limit of proportionality (elastic limit), the strain is proportional to stress. The ratio of these two is a constant and is called elastic modulus. The value of this modulus depends on the nature of strain produced. Within elastic limit the ratio of tensile stress to corresponding strain is called young modulus (Y). The ratio of volume stress to the volume strain is called bulk modulus and ratio of shearing stress to shearing strain is called modulus of rigidity. When a body elongates (or contracts) ~~lateraly~~ freely in the direction of a tensible force, it contracts (or elongates) lateraly i.e in direction ⊥ to the force. The ratio of the lateral strain to the longitudinal strain is called Poisson ratio.

Relation between Y, K, η & σ:-

Let a force P acts normally outwards, be applied to ends face of unit cube. If the force P is acting along the x axis be considered.



If 'e' be the extensions produced along the direction then as a unit cube is taken P and e indicates the stress and strain along the axis, if Y be the Young's modulus $Y = \frac{P}{e}$ or $e = \frac{P}{Y}$

Again σ be the poisson ratio then contractions in the other two direction along y and z axis are $\sigma e = \frac{\sigma P}{Y}$ which can be taken as equivalent to an extension $\frac{-\sigma P}{Y}$, the force P acting parallel to x axis produces extension $\frac{P}{Y}$, $\frac{-\sigma P}{Y}$ and $\frac{-\sigma P}{Y}$ along x axis, y axis & z axis. Similarly, the force P asking parallel to OY produces along the above axes extension $\frac{-\sigma P}{Y}$, $\frac{P}{Y}$, $\frac{-\sigma P}{Y}$ and the force parallel to OZ extends $\frac{-\sigma P}{Y}$, $\frac{-\sigma P}{Y}$ & $\frac{P}{Y}$

Thus the total extension along each ~~of the~~ the axes $= \frac{P}{Y} - \frac{2\sigma P}{Y} = \frac{P}{Y}(1-2\sigma)$ when the unit cube is simultaneously acted on by tensile force on ~~the~~ its faces. All the forces acting togather produces a volume stress of magnitude P which produces a volume strain $= \frac{P}{K}$. The volume strain is equal to 3 times the longitudinal strain along each direction.

$\therefore \frac{P}{K} = 3 \times$ longitudinal strain $= 3 \times$ extension along each of the axes

$$= \frac{3P}{Y}(1-2\sigma)$$

$$\therefore Y = 3K(1-2\sigma) \quad \text{——— (1)}$$

Now ~~if~~ ~~to~~ it is supposed that σ tensile stress 'p' acts along y axis, i.e on faces efhg and oxgz and that of compressing stress P acts on faces fxgh & ezoy, the net extension along x, y & z axis will be

$$\left(-\frac{P}{Y}-\frac{\sigma P}{Y}\right),\left(\frac{P}{Y}+\frac{\sigma P}{Y}\right) \& \left(-\frac{\sigma P}{Y}+\frac{\sigma P}{Y}\right)$$

The net result of applying such tensile and compressional faces together is a contraction $\left\{\frac{P}{Y}(1+\sigma)\right\}$ along x-axis and elongation $\left\{\frac{P}{Y}(1+\sigma)\right\}$ along x-axis

This elongation and contraction together may be looked upon a equivalent to shear θ in the x-y plane. The magnitude of this shear is equal to

$$\theta : \frac{2P}{Y}(1+\sigma)$$

$$\text{Hence } \eta = \frac{P}{\theta} = \frac{PY}{2P(1+\sigma)} = \frac{Y}{2(1+\sigma)} \quad \text{———— (2)}$$

$$\text{or } Y = 2\eta(1+\sigma) \quad \text{———— (3)}$$

Combing (1) & (3)

$$\frac{9}{Y} = \frac{3}{\eta}+\frac{1}{K}$$

$$\therefore Y = \frac{9\eta K}{3K+\eta} \quad \& \quad \sigma = \frac{3K-2\eta}{6K+2\eta}$$

These are the relation b/w the elastic constant.

Since $2\eta(1+\sigma) = 3K(1-2\sigma)$ $\therefore \frac{1+\sigma}{1-2\sigma} = \frac{3K}{2\eta}$ = is a +ve quantity since K & η are (+ve)

$\therefore \frac{1+\sigma}{1-2\sigma}$ is always positive.

But $\sigma < -1$, $\frac{1+\sigma}{1-2\sigma}$ becomes (−ve) & when $\sigma > \frac{1}{2}$, $\frac{1+\sigma}{1-2\sigma}$ is (−ve). hence σ must lies $\frac{1}{2}$ and −1.